2007 ACCA Calculus Competition

Multiple Choice:

1. If $f(x)$ is a polynomial of degree 5, what is the maximum number of inflection points $f(x)$ can have?

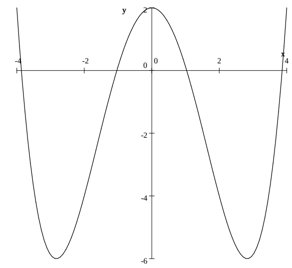
   A) 1 B) 2 C) 3 D) 4 E) 5

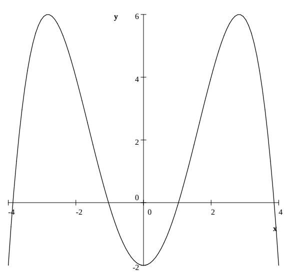
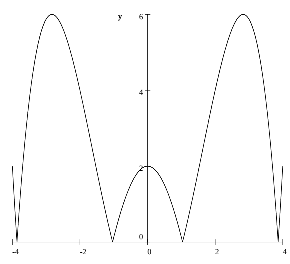
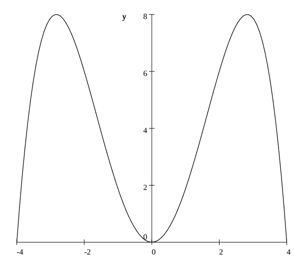
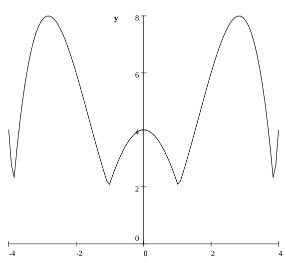
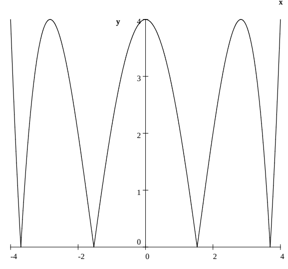
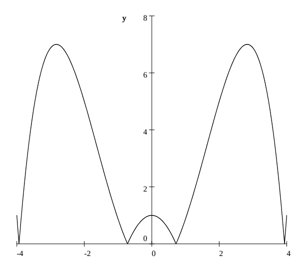
2. Which one of the functions below is continuous at $x = 0$ but *not* differentiable at $x = 0$.

   A) $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$ B) $f(x) = x^2 \sin\left(\dfrac{1}{x}\right)$ C) $f(x) = \begin{cases} x^2, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$

   D) $f(x) = \begin{cases} x^3 \cos\left(\dfrac{1}{x^2}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ E) $f(x) = |x - 1|$

3. Given the graph of $y = f(x)$ to the right, which graph represents $y = |f(x) + 2|$?



A) 
B) 
C) 
D) 
E) 
F) 

4. Find the maximum *slope* of $y = -2x^3 + 12x^2 - 10$.

   A) 22 B) 54 C) -10 D) 24 E) None of these

5. Which of the following are true statements?

   I) If $f + g$ is integrable on $[a, b]$, then f and g are both integrable on $[a, b]$.

   II) If f is integrable on $[a, b]$, then $\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$.

   III) If f is continuous on $[a, b]$ and $\int_a^b f(x)\,dx > 0$, then $f(c) > 0$ for some c$\in [a, b]$.

   IV) If f and g are continuous on $[a, b]$ and $\int_a^b f(x)\,dx = \int_a^b g(x)\,dx$, then $f(x) = g(x)$ for all $x \in [a, b]$.

   A) II B) I & II C) II & III D) I,II & III E) I,II & IV

6. Let n be an integer. Compute $\lim\limits_{x \to 0} \dfrac{1}{x^{4n}} \int_0^{x^n} \sin(t^3)\,dt$

   A) n^3 B) $\dfrac{1}{4}$ C) ∞ D) 0 E) $4n$

7. What integral represents the volume of the solid of revolution formed by revolving the region enclosed by $y = x$ and $y = x^2$ around the $x-$axis?

A) $\int_0^1 2\pi \left(\sqrt{y} - y\right)^2 dy$ B) $\int_0^1 \pi \left(x - x^2\right)^2 dx$ C) $\int_0^1 2\pi y \left(y - \sqrt{y}\right) dy$

D) $\int_0^1 \pi \left(x - \sqrt{x}\right) dx$ E) $\int_0^1 \pi \left(x^2 - x^4\right) dx$

8. Compute the derivative of the function $y = x^{x^x}$

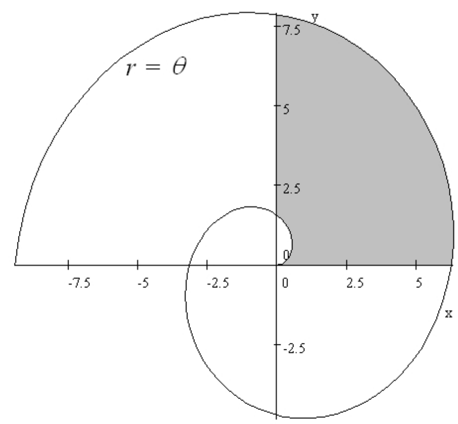
A) $(2x \ln x + x)\, x^{x^x}$ B) $\left(1 + \ln x + \dfrac{1}{x \ln x}\right) x^x \cdot \ln x \cdot x^{x^x}$ C) $x^x \cdot x^{x^x - 1} \cdot x^{x-1}$

D) $\left(x^x \ln x + x^{x-1}\right) x^{x^x}$ E) $\left(1 + \ln x + \dfrac{1}{\ln x}\right) \cdot x^{x^x}$

9. Find the surface area of the portion of the unit sphere between the planes $x = 1/4$ and $x = 1/2$.

A) $\dfrac{9}{16}\pi - \sqrt{2}\pi$ B) $\pi/2$ C) $\dfrac{1}{3}\pi^2 - 2\pi \arcsin \dfrac{1}{4}$ D) $\pi\left(\dfrac{\sqrt{15}}{2} - \sqrt{3}\right)$ E) None of these

10. Find the shaded area in the graph to the right:



A) $\frac{5}{4}\pi^3$ B) $\dfrac{1}{4}\pi^2$ C) $\frac{1}{8}\pi^3$ D) $\frac{3}{8}\pi$ E) 2π

11. The level curves of $z = x^2 + 2y^2$ are

A) circles B) straight lines C) Hyperbolas D) ellipses E) rectangles

12. Compute $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2 + y^4}$

A) $\dfrac{1}{4}$ B) 1 C) 0 D) $\dfrac{1}{2}$ E) Does not exist

13. Determine $\dfrac{dw}{dx}$ if $w = x^3 + \cos z$, $z = 3x^3y^2$, and $y = \sqrt{x^2 + 1}$.

A) $-6x^3y\left(x^2+1\right)^{1/2} \sin z$ B) $\left(3x^2\right)\left(9x^2y^2\right)\left(x^3 + x\right)$ C) $\left(3x^2 - \sin z\right)\left(18x^2y\right)\left(x\left(x^2+1\right)^{-1/2}\right)$

D) $3x^2 - \sin z\left(9x^2y^2 + 6x^4y\left(x^2+1\right)^{-1/2}\right)$ E) None of these.

14. The parametric curve given by $\left\langle -t^2, t^3 - 8t\right\rangle$, $t \in (-\infty, \infty)$ crosses itself at one point. Determine the slopes of the tangent lines at the point of crossing.

A) $\pm\sqrt{2}$ B) ± 1 C) $\pm 2\sqrt{2}$ D) $\pm\sqrt{6}$ E) ± 2

15. Evaluate $\int_C y dx - x dy$ where C is the the portion of the parabola $y = 1 - x^2$ from $(0, 1)$ to $(1, 0)$ followed by segment from $(1, 0)$ to $(2, 1)$.

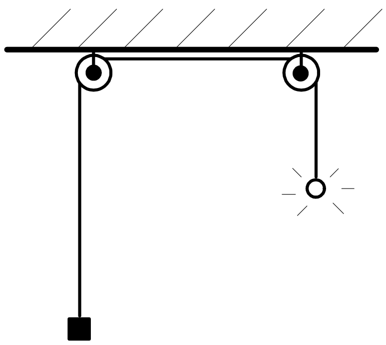
A) 1　　B) 1/3　　C) 0　　D) -1/2　　E) π.

Short Answer:

16. Compute $\lim\limits_{x \to 3^-} \dfrac{[\![x]\!] - 3}{x - 3}$

17. Let $F(x) = \int_0^x x e^{t^2} dt$. Compute $F''(x)$.

18. A rope 14 feet long is hanging over two pulleys that are attached to a horizontal beam and are 4 feet apart as pictured. The pulleys are 12 feet off the ground. A bright light is attached to one end of the rope and a weight is attached to the other end. If the weight causes the light to rise at a constant rate of 2 feet/sec, how fast is the shadow of the weight moving across the ground when the light is 10 feet off the ground?



19. Compute $\displaystyle\int_0^{\sqrt[5]{3}} \frac{x^4}{x^{10} + 9} dx$

20. Let (s_n) be the sequence defined recursively by $s_1 = \sqrt{5}$, $s_{n+1} = \sqrt{5 + s_n}$. Given that this sequence does have a limit, compute $\lim\limits_{n \to \infty} s_n$. You *must* state the *exact* value of the limit for credit.

21. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(3x - 2)^n}{n(-5)^n}$.

22. Compute the centroid of the region bounded by $y = x^2$ and $y = x^4$ and in the first quadrant.

23. A pilot in a plane whose path is described by the equations $x(t) = t^3 - 3$, $y(t) = t^2 + 1$, $z(t) = -t$, where t is time. At $t = 1$, the pilot fires a missle straight ahead , along a tangent line. The missle hits a target whose x coordinate is 7. Find the coordinates of the target.

24. Evaluate $\int_0^2 \int_{y/2}^1 e^{x^2} dx dy$.

25. Convert the following integral to spherical coordinates: $\int_0^1 \int_{-\sqrt{1-x^2}}^0 \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} (x^2 + y^2 + z^2)^{3/2} \, dz \, dy \, dx$. Do not evaluate.