2016 ACCA Calculus Competition

Multiple-Choice Questions

1. Assuming convergence, find $x = \sqrt{3 + \sqrt{3 + \sqrt{3 + \cdots}}}$

   (a) $\frac{1}{2}(\sqrt{5} + 1)$

   (b) $\frac{1}{2}(\sqrt{13} - 1)$

   (c) $\frac{1}{2}(\sqrt{5} - 1)$

   (d) $\frac{1}{2}(\sqrt{13} + 1)$

   (e) $\frac{1}{2}(\sqrt{13} - \sqrt{5})$

2. Calculate $\lim\limits_{x \to 0} \dfrac{\ln(1 - 3x) - \sin x}{1 - \cos^2 x}$

   (a) $-\frac{1}{2}$

   (b) $-\infty$

   (c) 0

   (d) DNE

   (e) ∞

3. Find $F'(x)$ for $F(x) = \displaystyle\int_1^{3x^2} \sqrt{4 + t^3}\, dt$.

   (a) $\sqrt{4 + 27x^6}$

   (b) $6x\sqrt{4 + x^3}$

   (c) $6x\sqrt{4 + 27x^6}$

   (d) $\frac{2}{3}(4 + 27x^6)^{\frac{3}{2}}$

   (e) $12x + 18\sqrt{3}\, x^4$

4. An oil rig is 16 miles from shore and 20 miles down-shore from a refinery. Land-based pipe costs \$60,000 per mile and underwater pipe costs \$100,000 per mile. Determine how the pipe should be laid to minimize the cost and calculate the minimum cost.

   (a) \$2,800,000
   (b) \$2,561,249.69
   (c) \$1,536,749.82
   (d) \$2,486,796.23
   (e) \$2,480,000

5. Express the volume of the wedge in the first octant that is cut from the cylinder $y^2 + z^2 = 1$ by the planes $y = x$ and $x = 1$ as a triple integral.

   (a) $\displaystyle\int_0^1 \int_0^{\sqrt{1-y^2}} \int_y^1 dxdzdy$
   (b) $\displaystyle\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_y^1 dxdzdy$
   (c) $\displaystyle\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_y^1 dxdzdy$
   (d) $\displaystyle\int_0^1 \int_x^1 \int_0^{\sqrt{1-y^2}} dzdydx$
   (e) $\displaystyle\int_{-1}^1 \int_x^1 \int_0^{\sqrt{1-y^2}} dzdydx$

6. Find the values of a, b, c which would make the function f below differentiable on (-1,1), given that $f(1) = f(-1)$:

$$f(x) = \begin{cases} x^2 + ax + b, & -1 \leq x < 0 \\ cx^2 + 4x + 1, & 0 \leq x \leq 1 \end{cases}$$

   (a) $a = 4, b = 1, c = -9$
   (b) $a = 4, b = 4, c = 4$
   (c) $a = 4, b = 1, c = -7$
   (d) $a = -4, b = 1, c = 1$
   (e) $a = 4, b = -1, c = 1$

7. Find the volume bounded by the paraboloid $z = 10 - 2x^2 - 2y^2$ and the plane $z = 2$.

(a) 8π
(b) 8
(c) 16π
(d) 0
(e) 4π

8. Evaluate the definite integral $\displaystyle\int_0^a \frac{x^2 + b^2}{x^2 + a^2}\, dx$.

(a) $\dfrac{(4-\pi)a + \pi b^2}{4a}$
(b) $4a^2 + \pi(b^2 - a^2)$
(c) $\dfrac{(2-\pi)a + \pi b^2}{2a}$
(d) $a + (b^2 - a^2)\ln 2$
(e) $\dfrac{(4-\pi)a^2 + \pi b^2}{4a}$

9. Let $g(x) = e^{-2x} f(3x)$, where f is a differentiable function, $f(0) = 4$ and $f'(0) = 5$. Find the value of $g'(0)$

(a) 7
(b) -30
(c) 9
(d) 5
(e) -10

10. Determine $m > 0$ so that the tangent line to the graph of

$$f(x) = \tan^{-1}\left(\frac{x}{m}\right) + \ln(x^2 + m^2)$$

at $x = 1$ is parallel to the line $x - 2y + 3 = 0$.

(a) $\frac{1}{2}$

(b) 3

(c) 1

(d) -1

(e) No such m exists.

11. Which of the following series are convergent?

I. $12 - 8 + \frac{16}{3} - \frac{32}{9} + \cdots$

II. $5 + \frac{5\sqrt{2}}{2} + \frac{5\sqrt{3}}{3} + \frac{5}{2} + \frac{5\sqrt{5}}{5} + \cdots$

III. $8 + 20 + 50 + 125 + \cdots$

(a) I only

(b) II only

(c) III only

(d) I and II

(e) II and III

12. The integral $\displaystyle\int_{-24}^{4} \frac{dx}{\sqrt[3]{(x-3)^2}}$

(a) converges to 4.

(b) converges to 12.

(c) converges to 9.

(d) diverges to $+\infty$.

(e) diverges to $-\infty$.

13. The radius of a right circular cone is increasing at a rate of 1.5 in/s and its height is decreasing at a rate of 3 in/s. Find the rate of change of the volume of the cone when the radius is 100 inches and the height is 150 inches. (*The volume of a cone is* $V = \frac{1}{3}\pi r^2 h$.)

(a) $10000\pi\,\text{in}^3$

(b) $5000\pi\,\text{in}^3$

(c) $15000\pi\,\text{in}^3$

(d) $\frac{40000}{3}\pi\,\text{in}^3$

(e) $22500\pi\,\text{in}^3$

14. Find the local maximum and minimum values and saddle points of the function $f(x, y) = x^2 + y^2 + x^2y + 4$.

(a) (0,0), $(\sqrt{2}, -1)$ and $(-\sqrt{2}, -1)$ are saddle points.

(b) $f(0, 0) = 4$ is a maximum value, and $f(\pm\sqrt{2}, -1) = 5$ is a maximum value.

(c) This function has no local extreme values

(d) $f(0, 0) = 4$ is a minimum value, and the points $(\pm\sqrt{2}, -1)$ are saddle points.

(e) $(0, 0)$ is a saddle point, and $f(\pm\sqrt{2}, -1) = 5$ is a minimum value.

15. Find the general term of the Maclaurin series for xe^{-x^2}.

(a) $\dfrac{(-1)^n x^{2n}}{(n+1)!}$

(b) $\dfrac{(-1)^n x^{2n+1}}{(n+1)!}$

(c) $\dfrac{(-1)^n x^{2n+1}}{n!}$

(d) $\dfrac{(-1)^{n+1} x^{2n+1}}{n!}$

(e) $\dfrac{(-1)^{n+1} x^{2n}}{n!}$

Short Answer Questions

1. Find the volume of the solid of revolution obtained by revolving the region bounded by the graphs of $x = y^2$ and $x = 1 - y^2$ about the line $x = -3$. Give an exact answer.

2. Find the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{(n+3)5^n}$.

3. Calculate $\displaystyle\lim_{h \to 0} \frac{\sqrt{x^2 + 2xh + h^2 - 5x - 5h} - \sqrt{x^2 - 5x}}{h}$.

4. Find the length of the curve $x(t) = e^t \cos t, y(t) = e^t \sin t$ for $0 \leq t \leq \pi$. Give an exact answer.

5. The temperature at each point (x, y, z) in a room is given by $T(x, y, z) = 4x^2 - 3y^2 + 5xyz$. A fly is hovering at the point $(1, 2, 4)$. In the direction of which vector should the fly move in order to cool off as quickly as possible?

6. Find a polynomial function f of minimum degree such that f has a maximum of 6 at $x = 1$ and a minimum of 2 at $x = 3$.

7. An elastic square is inscribed in a circle whose area is expanding at a rate of 16π square-inches per second. What is the rate of change of area for the inscribed square?

8. Evaluate the vector line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = (2xz + \sin y)\mathbf{i} + x \cos y\mathbf{j} + x^2\mathbf{k}$ and C is the curve parametrized by $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ for $0 \leq t \leq 2\pi$.

9. Calculate $\displaystyle\int_{1/e}^{e} \frac{\sqrt[3]{1 + \ln x}}{x}\, dx$.

10. Find the plane tangent to the surface $x^2 + 2y^2 + 3zx = 10$ at the point $(1, 2, \frac{1}{3})$.