2006 ACCA Calculus Exam

Part 1: Multiple Choice

#1 If g is a differentiable function and $f(x) = g(g(x))$, then $f''(x) =$

(A) $2g''(x) \cdot g(x) + 2[g'(x)]^2$
(B) $g''(g(x)) \cdot [g'(x)]^2 + g'(g(x)) \cdot g''(x)$
(C) $g''(g'(x)) \cdot g'(x)$
(D) $2[g'(x)]^2 + 2g''(g(x))$
(E) $g'(g'(x)) \cdot g''(x) + [g'(x)]^2$

#2 Let $f(x)$ be a differentiable function whose domain is the set of all real numbers. Suppose that $f(0) = -3$ and $f(10) = 7$. Which of the following statements are true?

I. There is a number c in the interval (0,10) such that $f(c) = 0$.
II. There is a number c in the interval [0,10] such that $f(c) \geq f(a)$ for every number a in the interval [0,10].
III. There is a number c in the interval (0,10) such that $f'(c) = 1$.

(A) I and II only.
(B) I and III only.
(C) II and III only.
(D) I, II, and III.
(E) None of the above.

#3 Find the value of $\lim\limits_{x \to \infty} \dfrac{1}{\sqrt{x+1} - \sqrt{x}}$.

(A) 1
(B) $\dfrac{1}{2}$
(C) 2
(D) ∞
(E) None of the above.

#4 The asymptote of the function $y = x(1 + \frac{1}{x})^x$ (defined on $x > 0$) is

(A) $x = 0$
(B) $y = 0$
(C) $y = ex - \dfrac{e}{2}$
(D) $y = \dfrac{e}{2}x + 1$
(E) None of the above.

$\boxed{\textbf{\#5}}$ The product of the *x*-coordinates of the points on the curve $x^4 + y^2 - 4x^2y + 27 = 0$ where the tangent line is vertical is

(A) -3 (B) 36 (C) 2 (D) -8 (E) None of the above.

$\boxed{\textbf{\#6}}$ Rounded to the nearest degree, what is the angle between the tangent vector to $f(t) = \left\langle 2t,\, \sin t,\, e^t \right\rangle$ at $t = 0$ and the positive *z* axis?

(A) 53° (B) 87° (C) 95° (D) 66° (E) None of the above.

$\boxed{\textbf{\#7}}$ $\lim\limits_{n\to\infty} \dfrac{\sqrt[n]{e} + \sqrt[n]{e^2} + \cdots + \sqrt[n]{e^{n-1}} + e}{n} =$

(A) $e - 1$ (B) 0 (C) e (D) 1 (E) None of the above.

$\boxed{\textbf{\#8}}$ $\lim\limits_{n\to\infty} \dfrac{e^n}{n!} =$

(A) 0 (B) 1 (C) e (D) ∞ (E) None of the above.

$\boxed{\textbf{\#9}}$ If A_1 is the area within the limaçon $r = 5 + 3\cos\theta$ and A_2 is the area within the limaçon $r = 20 + 12\cos\theta$, then the ratio A_2/A_1 equals

(A) 2 (B) 4 (C) 8 (D) 16 (E) None of the above.

$\boxed{\textbf{\#10}}$ **Problem:** $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{1/x^2} =$

(A) 0 (B) 1 (C) e (D) ∞ (E) None of the above.

$\boxed{\textbf{\#11}}$ For what set of values of x does $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converge conditionally?

(A) [-1, 0) (B) -1 only (C) -1 and 1 only
(D) [-1, 1) (E) None of the above.

$\boxed{\textbf{\#12}}$ If $p(x) = ax^3 + bx^2 + cx + d$, then $\int_0^{\infty} p(x)e^{-x}dx =$

(A) 0 (B) $a - b + c - d$ (C) $\frac{1}{4}a + \frac{1}{3}b + \frac{1}{2}c + d$
(D) $d + c + 2b + 6a$ (E) None of the above.

$\boxed{\textbf{\#13}}$ What is the length of the plane curve defined by $x(t) = t^2 + 1$ and $y(t) = t^3 - 2$, for $0 \le t \le 2$?

(A) $\frac{14}{3}$ (B) $\frac{24}{5}$ (C) $\frac{8}{27}(10\sqrt{10} - 1)$
(D) $\frac{4}{9}(10\sqrt{10} - 1)$ (E) None of the above.

#14 $\int_0^1 \int_w^1 \frac{1}{1+z^2}\, dz\, dw =$

(A) $1 - \ln 2$ (B) $\frac{1}{2}\ln 2$ (C) $\frac{\pi}{4}$

(D) $\frac{1}{2} + \ln 2$ (E) None of the above.

#15 Let S be the closed region in the xy-plane whose boundary is the parallelogram with vertices $(0,0)$, $(2,0)$, $(3,1)$, and $(1,1)$. The volume of the solid which lies under the surface $z = 2 + 4xy$ and over the region S is

(A) 8 (B) $\frac{28}{3}$ (C) $\frac{32}{3}$ (D) $\frac{73}{6}$ (E) None of the above.

Part 2: Free Response

$\boxed{\textbf{\#16}}$ If $f(\frac{y}{x}) = \dfrac{\sqrt{x^2+y^2}}{x}$ $(x>0)$, find $f(x)$.

$\boxed{\textbf{\#17}}$ Find the sum of the reciprocals of all the positive integers divisible by only powers of 2 and 5. In other words, compute $\dfrac{1}{1}+\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{5}+\dfrac{1}{8}+\dfrac{1}{10}+\ldots$.

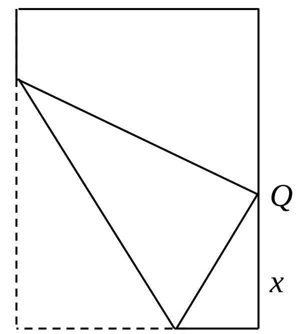
$\boxed{\textbf{\#18}}$ Find the volume of the solid obtained by revolving around the x axis the region $R=\{(x,y)\ |-1\le x\le 1,\quad 0\le y\le e^x+e^{-x}\}$.

$\boxed{\textbf{\#19}}$ Let $u = x^{y/z}$. Find the gradient of u at the point $(e^2, 5, -1)$.

$\boxed{\textbf{\#20}}$ Consider the graph of $y=x^3$. Suppose we start at the point (a,a^3), $a>0$, and find the equation of the tangent line to the graph at this point. This line will intersect the graph of $y=x^3$ at a second point – call it $(a_1, {a_1}^3)$. If we find the tangent line at this point, it again intersects the curve at another point $(a_2, {a_2}^3)$. If we iterate this process, find the sum of the infinite series $\displaystyle\sum_{k=1}^{\infty}\frac{1}{{a_k}^2}$.

$\boxed{\textbf{\#21}}$ Find the exact value of c so that the area of the region bounded by $y=\cos x$ and the x-axis for $0\le x\le c$ is exactly π.

#22 A rectangular piece of $8.5'' \times 11''$ paper (aligned with $8.5''$ at the bottom) is folded so that the lower left hand corner is at point Q along the right hand edge. If point Q is x inches above the lower right hand corner, find the value of x which minimizes the length of the crease.



#23 Let k be a positive real number. Find the maximum area of a triangle in Quadrant I which can be formed by the coordinate axes and a tangent line to the curve $y = \frac{k}{x}$. Express your answer in terms of k.

#24 Find the volume of the solid bounded by the surface $z = 4 - y^2$, the plane $z = 4 - x$, and the xy-plane.

#25 For each real number a, define $f_a(x)$ to be equal to the smaller of the two values $(x-a)^2$ and $(x-a-2)^2$. Let $F(a) = \int_0^1 f_a(x)\, dx$. Find the maximum value of $F(a)$ with $-2 \le a \le 2$.