2011 ACCA CALCULUS COMPETITION

MULTIPLE-CHOICE QUESTIONS

1. A particle is moving along a straight line so that its velocity at time $t \geq 0$ is $v(t) = 3t^2$. At what time t during the interval from $t = 0$ to $t = 9$ is its velocity the same as the average velocity over the entire interval $0 \leq t \leq 9$?

(A) 3 (B) 4.5 (C) $3\sqrt{3}$ (D) $\frac{9}{2}\sqrt{2}$ (E) $9\sqrt{3}$

2. Suppose f is continuous and $x^2 \leq f(x) \leq 6$ for all x in the interval [-1,2]. Find values of A and B such that $A \leq \int_{-1}^{2} f(x)\,dx \leq B$.

(A) $A = 3, B = 11$ (B) $A = 4,\ B = 18$ (C) $A = 3,\ B = 18$
(D) $A = 4, B = 11$ (E) None of the above.

3. Find the equation of the line which is normal to the curve $f(x) = 3x^2 - 6x - 2$ and passes through the point $(2, -2)$.

(A) $y = \frac{x}{6} - \frac{7}{3}$ (B) $y = -\frac{x}{6} - \frac{5}{3}$ (C) $y = 6x - 14$
(D) $y = -6x + 10$ (E) No such line exists.

4. Find the area of the region bounded by the lines $\vartheta = 0$ and $\vartheta = \frac{\pi}{2}$ and by the polar curve $r = e^{\vartheta}$ for $0 \leq \vartheta \leq \frac{\pi}{2}$.

(A) $\frac{1}{2}\left(e^{\pi/2} - 1\right)$ (B) $\frac{1}{2}(e^{\pi} - 1)$ (C) $\frac{1}{4}e^{\pi}$ (D) $\frac{1}{4}\left(e^{\pi/2} - 1\right)$
(E) $\frac{1}{4}(e^{\pi} - 1)$

5. Find the approximate arc length of the curve with equation $y = x^{3/2}$ from $x = 0$ to $x = 5$.

(A) 8.92 (B) 12.41 (C) 16.18 (D) 23.75 (E) 33.13

6. A bird drops a coconut from a height of 100 ft. Assuming that the coconut falls only under the influence of gravity, find the speed (in feet/sec) at which it hits the ground.

(A) 9.8 (B) 16 (C) 32 (D) 80 (E) 100

7. Which of the following is NOT true about $f(x) = \cos(-x + \pi)$?

(A) f has the same period as $\sin(x - \pi)$.
(B) f has one inflection point in $(-\pi, \pi)$.
(C) $\frac{d^2f}{dx^2} + f = 0$.
(D) f has a minimum at $x = 0$.
(E) f has one critical point in $(-\pi, \pi)$.

8. Find the area of the largest rectangle that can be inscribed in the region bounded by $y = 3 - x^2$ and the x-axis.

(A) 1 (B) 2 (C) $\frac{9}{4}$ (D) 4 (E) $4\sqrt{3}$

9. If $f(x)$ and $g(x)$ are continuous functions on the interval $[a, b]$, and $\int_a^b f(x)\,dx = \int_a^b g(x)\,dx$, which one of the following MUST be true?

(A) $f(x) = g(x)$
(B) $\frac{d}{dx}\int_a^x f(t)\,dt = \frac{d}{dx}\int_a^x g(t)\,dt$.
(C) $f(x)$ and $g(x)$ have the same average value on $[a, b]$
(D) $f(x) = g(x) + C$ for some constant C
(E) None of the above.

10. Find the maximum value of $f(x, y) = 2x + y^2$ on the circle $x^2 + y^2 = 9$

(A) 0 (B) 1 (C) 6 (D) 9 (E) None of the above.

11. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}(2x - 4)^n$.

(A) $\left(\frac{3}{2}, \frac{5}{2}\right)$ (B) $\left(\frac{3}{2}, \frac{5}{2}\right]$ (C) $\left(-\frac{1}{2}, \frac{1}{2}\right)$
(D) $\left(-\frac{1}{2}, \frac{1}{2}\right]$ (E) $(-\infty, \infty)$

12. Find the limit of the sequence $\{a_n\}$ defined by $a_1 = 1$, and $a_n = 1 - \frac{1}{2}a_{n-1}$ for $n \geq 2$.

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) The limit does not exist.

13. Which one of the following functions is continuous everywhere AND has at least one point where it is not differentiable?

(A) $\tan x$ (B) $\frac{|x|}{x}$ (C) $\sin x$
(D) e^{-x} (E) $|x^3 + x|$

14. Which of the following integrals represents the following integral with the order of integration reversed? $\int_0^3 \int_{e^x}^{e^3} f(x,y)\,dy\,dx$

(A) $\int_1^3 \int_{\ln y}^{e^3} f(x,y)\,dx\,dy$
(B) $\int_{e^x}^3 \int_0^{e^3} f(x,y)\,dx\,dy$
(C) $\int_0^1 \int_{e^x}^3 f(x,y)\,dx\,dy$
(D) $\int_1^{e^3} \int_0^{\ln y} f(x,y)\,dx\,dy$
(E) $\int_0^3 \int_0^{\ln y} f(x,y)\,dx\,dy$

15. Let C be the curve defined by $x = t^2 + t + 1$ and $y = t^3 - t - 1$. Determine the x-intercept for the line tangent to C at $(3, -1)$.

(A) $-\frac{11}{2}$ (B) -3 (C) 1 (D) $\frac{11}{3}$ (E) $\frac{9}{2}$

SHORT-ANSWER QUESTIONS

16. Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{n}{a^n}$$

where a is a positive constant.

17. A large spherical snowball melts so that its surface area decreases at the constant rate of 8 cm^2 per minute. Find the rate at which the volume is changing when the radius is 10 cm.

18. Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 1$ and whose cross-sections are squares (perpendicular to the base).

19. Suppose the line tangent to the curve $y = f(x)$ at $x = 3$ passes through the points $(-2,3)$ and $(4,-1)$. Find $f(3)$ and $f'(3)$.

20. Find the values of the constants m and b that would make the following function differentiable at $x = 1$: $f(x) = \begin{cases} x^2 - 5 & if\ x < 1 \\ mx + b & if\ x \geq 1. \end{cases}$

21. Find the value of $\iint_R xy\, dA$, where R is the region in the first quadrant bounded by the curve with equation $x^2 + y^2 = 16$.

22. Find an equation of the line tangent to the curve with equation $y = 6x^3 + 36x^2 + 24x$ at its point of inflection. Write your equation in the form $y = mx + b$.

23. Let S be the solid in the first octant bounded by the planes $x = 0, y = 0, z = 0$, and the plane tangent to $z = 11 - x^2 - 4y^2$ at the point where $x = 1$ and $y = 1$. Find the volume of S.

24. Determine the value of k so that the area of the region below the graph of $y = -x + 8$ and above the x-axis, between $x = -k$ and $x = k$, is 80.

25. Find the x-coordinate(s) of the point(s) where the line tangent to the curve with equation $x^4 + y^4 - 4y = 13$ is vertical.