2012 ACCA CALCULUS COMPETITION

MULTIPLE-CHOICE QUESTIONS

1. Let $f(x) = \cos 2x$. Find $f^{(2012)}(0)$, where $f^{(n)}(x)$ denotes the $n-$th derivative of $f(x)$.

   (A) 0 (B) 2^{2012} (C) 2^{2011} (D) -2^{2012} (E) -2^{2011}

2. Find the equation of the tangent line to the curve $y^2 \sin x + x^2 \cos y = 0$ at the point $(\pi, \pi/2)$.

   (A) $y = \frac{1}{4}x + \frac{\pi}{4}$ (B) $y = \frac{1}{4}x + \frac{\pi}{2}$ (C) $y = -\frac{1}{4}x + \frac{\pi}{4}$
   (D) $y = -\frac{1}{4}x + \frac{3\pi}{4}$ (E) $y = -\frac{1}{2}x + \pi$

3. Calculate the area of the region bounded by the curves $y = \dfrac{x}{x^2+1}$ and $y = \dfrac{x^2}{2}$.

   (A) 1 (B) $\dfrac{1}{2}\ln 2 - \dfrac{1}{6}$ (C) $\dfrac{1}{6}\ln 2 - \dfrac{1}{2}$ (D) $\dfrac{1}{2}\ln 6 - \dfrac{1}{6}$ (E) $\dfrac{1}{6}\ln 6 - \dfrac{1}{2}$

4. A 10 ft. long ladder rests against the side of a vertical wall. If the bottom of the ladder slides away from the wall at a rate of $2\frac{\text{ft}}{\text{s}}$, at what rate is the top of the ladder sliding down the wall when it is 2 ft above the ground?

   (A) $-4\sqrt{3}\frac{\text{ft}}{\text{s}}$ (B) $-2\sqrt{3}\frac{\text{ft}}{\text{s}}$ (C) $-4\sqrt{2}\frac{\text{ft}}{\text{s}}$ (D) $-4\sqrt{6}\frac{\text{ft}}{\text{s}}$ (E) $-8\sqrt{6}\frac{\text{ft}}{\text{s}}$

5. Which of the following statements would allow you to conclude that $\sum_{n=1}^{\infty} a_n$ is divergent?

   (A) $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$.

   (B) $a_n \geq \dfrac{1}{n^2}$ for all n.

   (C) $\lim_{n\to\infty} a_n = 1$.

   (D) $\lim_{n\to\infty} \dfrac{a_{n+1}}{a_n} = 1$.

   (E) $a_n \leq \dfrac{1}{\sqrt{n}}$ for all n.

6. Find the volume of the solid obtained by rotating the region bounded by $y = x^2 + 1$, $x = 0$, and $y = 2$ about the line $y = 2$.

(A) $\frac{4\pi}{5}$ (B) $\frac{8\pi}{15}$ (C) $\frac{12\pi}{5}$ (D) $\frac{4\pi}{15}$ (E) $\frac{2\pi}{5}$

7. Suppose that f and g are differentiable functions and $g(x) = xf(x^2 + 4)$. If $f(8) = 20$ and $f'(8) = -3$, what is $g'(2)$?

(A) -4 (B) -6 (C) -24 (D) 8 (E) 14

8. Suppose that f and f' are continuous on $[0, \ln 3]$ and that $f(0) = 0$, $f(\ln 3) = 4$, and $\int_0^{\ln 3} e^{2x} f'(x)\, dx = 10$. Find the value of $\int_0^{\ln 3} e^{2x} f(x)\, dx$.

(A) 13 (B) -13 (C) 0 (D) 5 (E) -5

9. Evaluate $\lim_{x \to 0} \frac{\sin 2x}{(1+x)\ln(1+x)}$.

(A) -2 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 2

10. Let $A, B,$ and C be the points of intersection of the ellipsoid $4x^2+4y^2+9z^2 = 36$ with the positive directions of the $x-$axis, $y-$axis, and $z-$axis, respectively. Where does the plane through the points $A, B,$ and C intersect the line given parametrically by $x = t$, $y = t$, $z = t$?

(A) $(1, 1, 1)$ (B) $(\frac{6}{7}, \frac{6}{7}, \frac{6}{7})$ (C) $(0, 0, 0)$ (D) $(2, 3, 3)$ (E) $(\frac{2}{3}, \frac{2}{3}, \frac{2}{3})$

11. $\int_1^5 \int_0^{\ln x} f(x, y)\, dydx$ is equivalent to which of the following?

(A) $\int_1^5 \int_0^{\ln x} f(x, y)\, dxdy$ (B) $\int_0^{e^5} \int_{e^y}^5 f(x, y)\, dxdy$ (C) $\int_0^{\ln 5} \int_{e^y}^5 f(x, y)\, dxdy$

(D) $\int_0^{\ln x} \int_1^5 f(x, y)\, dxdy$ (E) $\int_1^{e^5} \int_0^{e^y} f(x, y)\, dxdy$

12. Find the absolute maximum M and minimum m of the function $f(x, y) = x^2 + 2y^2 - 2x$ on the closed disc $x^2 + y^2 \leq 4$.

(A) $M = 1, m = 1$ (B) $M = 9, m = 0$ (C) $M = 9, m = -1$
(D) $M = 8, m = -8$ (E) $M = 9, m = -9$

13. Determine all real numbers r for which the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^r + \ln n}$ converges.

(A) $r > -1$ (B) $r > 0$ (C) $r > \frac{1}{2}$ (D) $r > 1$ (E) $r > \frac{3}{2}$

14. Suppose $f(x, y, z) = ye^{xz}$. A unit vector $\mathbf{v}$ in the direction of fastest decrease of f at the point $(1, 1, 0)$ is

(A) $-\mathbf{i}$ (B) $\mathbf{j}+\mathbf{k}$ (C) $-\mathbf{j}-\mathbf{k}$ (D) $\frac{1}{\sqrt{2}}(\mathbf{j}+\mathbf{k})$ (E) $-\frac{1}{\sqrt{2}}(\mathbf{j}+\mathbf{k})$

15. Let $F(x) = \int_0^x e^{-t^2}\, dt$. The first four non-zero terms of the Maclaurin series for $F(x)$ are:

(A) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ (B) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!}$ (C) $x - \frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7 \cdot 3!}$

(D) $x - \frac{x^3}{3} - \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7 \cdot 3!}$ (E) $1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!}$

SHORT-ANSWER QUESTIONS

16. Find all points on the graph of $y = x^2$ such that the tangent line to the curve at these points passes through $(0, -2)$.

17. Find all critical numbers for the function $f(x) = \sqrt[3]{9 - x^2}$.

18. Find the interval of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2n+1}$.

19. Suppose f is continuous and $f(-x) + f(x) = x^2$. Find $\displaystyle\int_{-1}^{1} f(x)\, dx$.

20. Suppose that $\displaystyle\sum_{n=2}^{\infty} \left(\frac{1}{c}\right)^n = 3$. What is c?

21. Find the area of the largest rectangle that can be inscribed in a semicircle with radius 4.

22. Evaluate $\displaystyle\lim_{h \to 0} \frac{\int_1^{3+h} \sin(t^2)\, dt - \int_1^3 \sin(t^2)\, dt}{h}$.

23. Let $f(x) = \displaystyle\int_0^x e^{-t^2}\, dt$. Given that $\displaystyle\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$, evaluate $\displaystyle\int_0^{\infty} e^{-x^2 + f(x)}\, dx$.

24. Calculate $\displaystyle\iint_R y\, dA$ over the region in the first quadrant between the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.

25. Determine an equation of the plane that is tangent to the surface with equation $z = x^2 + y^2$ at the point $(3, 4, 25)$.

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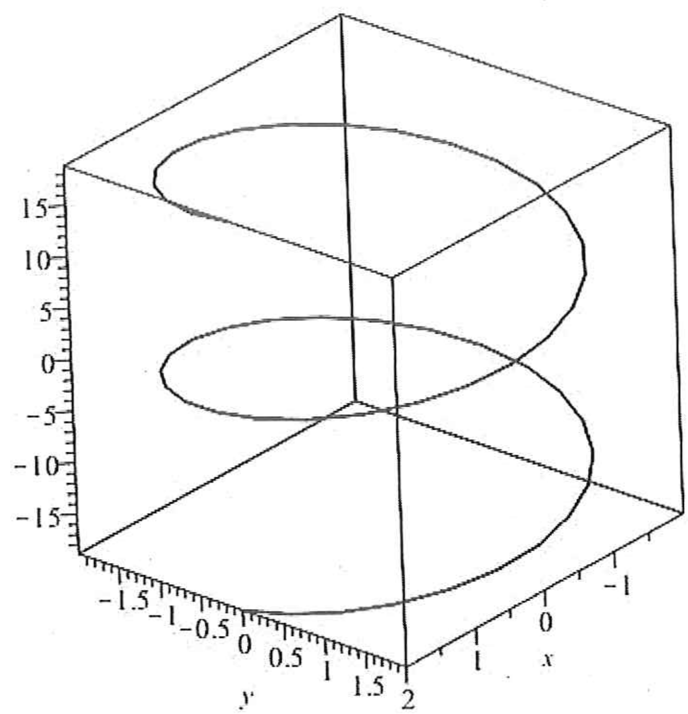
2 sides!

1. The curve $\mathbf{r}(t) = < 2\cos t, 2\sin t, 3t >$ is shown below. Find $\mathbf{T}, \mathbf{N}$, & $\mathbf{B}$. Sketch $\mathbf{T}, \mathbf{N}$, & $\mathbf{B}$ when $t = \dfrac{3\pi}{2}$.



2. Below is the graph of the space curve given by the vector valued equation $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} - \frac{1}{3}t^3\mathbf{k}$.

a) Based on an inspection of the graph, at what point is the curvature the greatest?

b) Find the curvature function $\kappa(t)$ for the space curve.

c) Verify analytically the point at which the curvature is the greatest.