2013 ACCA Calculus Competition

Multiple Choice Questions

1. Find the indefinite integral $\int 4\csc^2 x e^{\cot x}\,dx$.

   a. $-4e^{\cot x} + C$
   b. $-4\cot x \csc x + C$
   c. $-4\cot x + C$
   d. $e^{5\cot x} + C$
   e. $-4\csc x + C$

2. Solve the differential equation $\left(36 + \tan^2 x\right)y' = \sec^2 x$.

   a. $y = \frac{1}{6}\arctan\left(\frac{\sec x}{6}\right) + C$
   b. $y = \arctan\left(\frac{\sec x}{6}\right) + C$
   c. $y = \frac{1}{6}\arctan\left(\frac{\tan x \sec x}{6}\right) + C$
   d. $y = \frac{1}{6}\arctan\left(\frac{\tan x}{6}\right) + C$
   e. $y = \arctan\left(\frac{\tan x}{6}\right) + C$

3. Suppose a damping force affects the vibration of a spring so that the displacement of the string is given by $y = e^{-8t}(\cos 2t + 9\sin 2t)$. Find the average value of *y* on the interval from $t = 0$ to $t = \pi$. Round your answer to three decimal places.

   a. 0.406
   b. 0.131
   c. 0.122
   d. 0.129
   e. 0.382

4. Evaluate the limit $\lim\limits_{x \to 0^+} \left(e^x + 5x\right)^{5/x}$.

   a. e^5
   b. e^{10}
   c. 1
   d. e^{30}
   e. $e + 5$

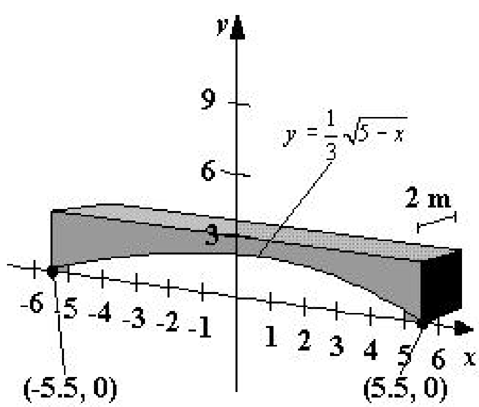
5. Consider the function given by $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-6)^n}{n}$. Find the interval of convergence for $f'(x)$.

a. $[-6, 6]$
b. $[5, 7]$
c. $(0, 7)$
d. $(-6, 6)$
e. $(5, 7)$

6. A solid is generated by revolving the region bounded by $y = \sqrt{100 - x^2}$ and $y = 0$ about the y-axis. A hole, centered along the axis of revolution, is drilled through this solid so that one-third of the volume is removed. Find the diameter of the hole. Round your answer to three decimal places.

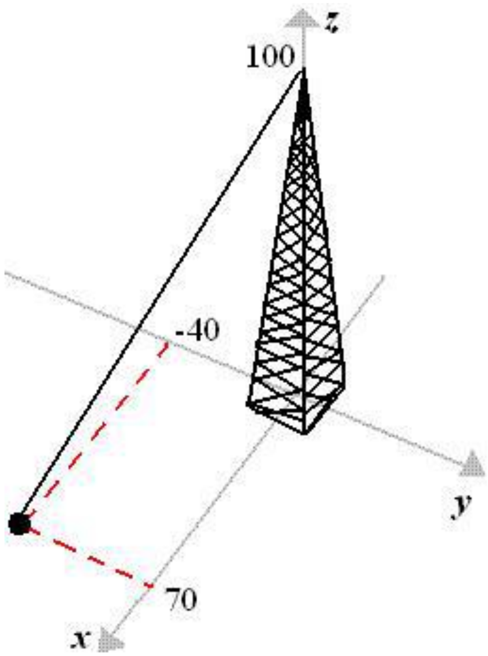
a. 11.547
b. 5.774
c. 4.867
d. 18.838
e. 9.734

7. Concrete sections for the new building have the dimensions (in meters) and shape as shown in the figure (the picture is not necessarily drawn to scale). One cubic meter of concrete weighs 4320 pounds. Find the weight of the section. Round your answer to the nearest pound.



a. 268,492 pounds
b. 263,654 pounds
c. 216,267 pounds
d. 242,187 pounds
e. 251,865 pounds

8. Suppose the guy wire to a 100-foot tower has a tension of 500 pounds. Using the distances shown in the figure, write the component form of the vector **F** representing the tension in the wire. Round numerical values in your answer to the nearest integer.



a. $\mathbf{F} = \langle 302, 186, 419 \rangle$

b. $\mathbf{F} = \langle 261, 145, 378 \rangle$

c. $\mathbf{F} = \langle 302, 186, 419 \rangle$

d. $\mathbf{F} = \langle 272, -156, -389 \rangle$

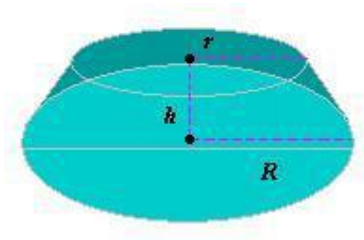
e. $\mathbf{F} = \langle 272, 156, 389 \rangle$

9. Find the volume of the parallelepiped with the following vertices.

$(0,0,0), (5,0,0), (0,9,3), (5,9,3), (2,0,4), (7,0,4), (2,9,7), (7,9,7)$

a. 180
b. 167
c. 60
d. 30
e. 22

10. The two radii of the frustum of a right circular cone are increasing at a rate of 4 centimeters per minute, and the height is increasing at a rate of 12 centimeters per minute (see figure). Find the rate at which the volume is changing when the two radii are 15 centimeters and 30 centimeters, and the height is 10 centimeters.



a. $7,275\pi\ \text{cm}^3/\text{min}$
b. $8,100\pi\ \text{cm}^3/\text{min}$
c. $8,250\pi\ \text{cm}^3/\text{min}$
d. $4,800\pi\ \text{cm}^3/\text{min}$
e. $7,650\pi\ \text{cm}^3/\text{min}$

11. Find the integral $\int \frac{e^x}{\left(e^{2x}+1\right)\left(e^x-3\right)}\,dx$.

a. $\int \frac{e^x}{\left(e^{2x}+3\right)\left(e^x-3\right)}\,dx = \frac{1}{24}\left(\arctan\left(e^x\right)\right)+C$

b. $\int \frac{e^x}{\left(e^{2x}+3\right)\left(e^x-3\right)}\,dx = \frac{1}{24}\left(\ln\left|e^{2x}+3\right|-2\arctan\left(e^x\right)\right)+C$

c. $\int \frac{e^x}{\left(e^{2x}+3\right)\left(e^x-3\right)}\,dx = \frac{1}{24}\left(2\ln\left|e^x-3\right|-\ln\left|e^{2x}+6\right|-4\arctan\left(e^{2x}\right)\right)+C$

d. $\int \frac{e^x}{\left(e^{2x}+3\right)\left(e^x-3\right)}\,dx = \frac{1}{20}\left(2\ln\left|e^x-3\right|-\ln\left|e^{2x}+1\right|-6\arctan\left(e^x\right)\right)+C$

e. $\int \frac{e^x}{\left(e^{2x}+3\right)\left(e^x-3\right)}\,dx = \frac{1}{24}\left(2\ln\left|e^{2x}-6\right|-\ln\left|e^{2x}+6\right|-2\arctan\left(e^x\right)\right)+C$

12. Evaluate the limit $\lim_{x \to 0^+} \left(e^x + 5x\right)^{5/x}$.

    a. e^5
    b. e^{10}
    c. 1
    d. e^{30}
    e. $e + 5$

13. Find the area of the surface for the portion of the sphere $x^2 + y^2 + z^2 = 25$ inside the cylinder $x^2 + y^2 = 16$.

    a. 80π
    b. 60π
    c. 13π
    d. 40π
    e. 20π

14. A buoy oscillates in simple harmonic motion $y = A\cos \omega t$ as waves move past it. The buoy moves a total of 3.5 feet (vertically) between its low point and its high point. It returns to its high point every 14 seconds. Write an equation describing the motion of the buoy if it is at its high point at *t* = 0. (g = 32 ft/s^2)

    a. $y = 3.5\cos\frac{\pi}{7}t$
    b. $y = 1.75\cos 7\pi t$
    c. $y = 1.75\cos\frac{\pi}{14}t$
    d. $y = 1.75\cos\frac{\pi}{7}t$
    e. $y = 3.5\cos 14\pi t$

15. The quarterback of a football team releases a pass at a height of 8 feet above the playing field, and the football is caught by a receiver 46 yards directly downfield at a height of 4 feet. The pass is released at an angle of 30° with the horizontal. Find the time in seconds the receiver has to reach the proper position after the quarterback releases the football. Round your answer to one decimal place. (g = 32 ft/s^2)

    a. 7.0 seconds
    b. 2.3 seconds
    c. 4.1 seconds
    d. 0.7 second
    e. 1.1 seconds

Short-Answer Questions

16. Consider the function $f(x) = \frac{-2+\sqrt{3+x^{\frac{1}{3}}}}{x-1}$. Calculate $\lim_{x\to 1} f(x)$.

17. A particle is moving along the graph of $= \sqrt[3]{x}$. When x = 8, the y-component of the position of the particle is increasing at the rate of 1 centimeter per second. How fast is the distance from the origin changing at this moment?

18. Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius 5 cm.

19. Suppose that $f(x) = \sin(\cos(\tan(x^2)))$. Determine $f'(x)$.

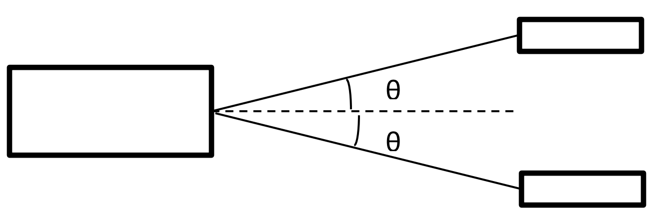
20. Determine the volume under the curve $f(x, y) = x^2 + 2y^2$ over the region R defined by the triangle in the xy-plane whose vertices are (0,0), (-1,2), and (1,2).

21. Determine $\frac{d}{dx}\int_{-\sqrt{x}}^{\sqrt{x}} \frac{1}{1+t^4}\, dt$.

22. A torus is formed by revolving the region bounded by the equation $(x-2)^2 + y^2 = 1$ about the y-axis. Calculate the volume of the torus.

23. Consider the power series $\sum_{n=0}^{\infty} a_n\, x^n = 1 + 2x + 3x^2 + x^3 + 2x^4 + 3x^5 + x^6 + \cdots$ Find the radius of convergence and the sum of this power series.

24. A loaded barge is being towed by two tugboats, and the magnitude of the resultant is 6000 pounds directed along the axis of the barge. Each towline makes an angle of 20° with the axis of the barge. Find the tension in the towlines.



25. Evaluate the integral $\int_0^{\infty}\int_0^{\infty} \frac{1}{(1+x^2+y^2)^2}\, dxdy$.