2014 ACCA Calculus Competition
Multiple Choice Questions

1). Suppose $h(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$, $f'(6) = 7$. Find the value of $h'(3)$.

a. 4
b. 8
c. 16
d. 28
e. 24

2). Let $f(x) = \displaystyle\int_{3x}^{2} \frac{sin(t)}{t^2}$, where $t > 0$. Find $f'(x)$.

a. $\frac{sin(3x)}{3x^2}$

b. $-\frac{sin(3x)}{3x^2}$

c. $\frac{cos(x)}{x^2}$

d. $-\frac{sin(3x)}{9x^2}$

e. $-\frac{cos(3x)}{9x^2}$

3). If $f_1 = 2$ and $f_{n+1} = f_n + \frac{1}{2}$ for all integers $n > 1$, then what is f_{101}.

a. 49
b. 50
c. 51
d. 52
e. 53

4). $\displaystyle\int_0^1 \int_0^{sin(y)} \frac{1}{\sqrt{1-x^2}}\ dx\ dy$=

a. $\frac{1}{3}$

b. $\frac{1}{2}$

c. $\frac{\pi}{4}$

d. 1

e. $\frac{\pi}{3}$

5). A lighthouse is on a small island 3 km away from the nearest point P on a straight shoreline, and its light makes 4 revolutions per minute. At what rate is the beam of light moving along the shoreline when it is 1 km away from P.

a. 20π km/min

b. $\frac{70\pi}{3}$ km/min

c. $\frac{80\pi}{3}$ km/ min

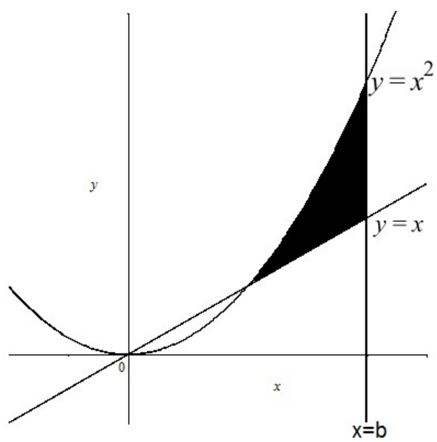
d. $\frac{100\pi}{3}$ km/min

e. $\frac{110\pi}{3}$ km/min

6). The function $f(x, y) = xy - x^3 - y^3$ has a relative maximum at the point

a. $(0, 0)$
b. $(1, 1)$
c. $(-1, -1)$
d. $(1, 3)$
e. $(\frac{1}{3}, \frac{1}{3})$

7). If $b > 0$ and if $\int_0^b x\,dx = \int_0^b x^2\,dx$, then the area of the shaded region in the figure below is



a. $\frac{1}{12}$

b. $\frac{1}{6}$

c. $\frac{1}{4}$

d. $\frac{1}{3}$

e. $\frac{1}{2}$

8). Find the length of the curve $x = 2t^2 - 1$ and $y = -4t^2 + 3$ for $0 \le t \le 1$.

a. $4\sqrt{5}$
b. $8\sqrt{5}$
c. $8\sqrt{3}$
d. $2\sqrt{3}$
e. $2\sqrt{5}$

9). $\displaystyle\int_0^1 \frac{x}{1+x^2}\, dx =$

a. 1

b. $\frac{\pi}{4}$

c. $tan^{-1}(\frac{\sqrt{2}}{2})$

d. $ln|2|$

e. $ln|\sqrt{2}|$

10).In the xy-plane, if C is the circle $x^2 + y^2 = 9$, oriented counterclockwise, then
$\displaystyle\oint_C -2x\ dx + x^2\ dy =$

a. 0
b. 6π
c. 9π
d. 12π
e. 18π

11). What is the area of the region bounded by the coordinate axes and the line tangent to the graph of $y = \frac{1}{8}x^2 + \frac{1}{2}x + 1$ at the point (0,1)?

a. $\frac{1}{16}$

b. $\frac{1}{8}$

c. $\frac{1}{4}$

d. 1

e. 2

12). Which of the following statements are true for every function f, defined on the set of real numbers, such that $\lim_{x\to 0} \frac{f(x)}{x}$ is a real number L and $f(0) = 0$?

I. f is differentiable at 0
II. $L = 0$
III. $\lim_{x\to 0} f(x) = 0$

a. None
b. I only
c. III only
d. I and III only
e. I, II, and III

13). All function f defined on the xy-plane such that $\frac{\partial f}{\partial x} = 2x + y$ and $\frac{\partial f}{\partial y} = x + 2y$ are given by $f(x, y) =$

a. $x^2 + xy + y^2 + C$
b. $x^2 - xy + y^2 + C$
c. $x^2 - xy - y^2 + C$
d. $x^2 + 2xy + y^2 + C$
e. $x^2 - 2xy + y^2 + C$

14). $\sum_{n=1}^{\infty} \frac{n}{n+1} =$

a. $\frac{1}{e}$
b. $ln(2)$
c. 1
d. e
e. $+\infty$

15). Find an equation for the plane consisting of all points that are equidistant from the two points (1,1,0) and (0,1,1)

a. $x - z = 0$
b. $x - z = 1$
c. $x - y = 0$
d. $x - y = 1$
e. $x + z = 0$

Short Answers

16). Find the point where the tangent to the curve $y = x\sqrt{3-x}$ has zero slope.

17). Evaluate the integral $\int ln(1+x^2)\ dx$.

18). Given that $f(x) = x^3 + ax^2 + bx$ has critical numbers at $x = 1$ and $x = 3$, find a and b.

19). Let f be the function defined by $f(x, y) = 5x - 4y$ on the region in the xy-plane satisfying the inequalities $x \leq 2, y \geq 0, x + y \geq 1$ and $y - x \leq 0$. Find the maximum value of f on this region.

20). In xyz-space, give an equation of the plane tangent to the surface $z = e^{-x} sin(y)$ at the point where $x = 0$ and $y = \frac{\pi}{2}$.

21). Consider the following two player game. Two players take turns tossing a fair die, i,e a 6 sided dice where each side has an equal probability of landing up. The first player to get a 5 wins the game. What is the probability that the player who starts first wins the game?

22). Determine the interval of convergence for the Taylor Series centered at zero (i.e the Maclaurin series) for $f(x) = ln(1 - x)$.

23). Calculate $\lim_{x \to \pi} \frac{e^{-\pi} - e^{-x}}{sin(x)}$.

24). In how many of the eight standard octants of xyz-space does the graph of $z = e^{x+y}$ appear?

25). Let $f(x) = \int_1^x \frac{1}{1+t^2}\, dt$ for all real x. Give the equation of the line tangent to the graph of f at the point $(2, f(2))$.