2005 ACCA CALCULUS COMPETITION QUESTIONS

Before beginning the exam, make sure every team member has read and understood all of the instructions provided on the Student Instruction Sheet.

PART ONE: MULTIPLE CHOICE QUESTIONS

1. Find the constants a and b such that the function $f(x) = \begin{cases} 2 & x \le -1 \\ ax+b & -1 < x \le 3 \\ x^2 - 2x - 5 & x \ge 3 \end{cases}$ is continuous on the entire real line.

   a. a = -1 and b = 1

   b. a = 1 and b = -1

   c. a = 2 and b = -2

   d. a = -2 and b = 2

   e. none of the above

2. Which of the following series converges absolutely?

   a. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{7\sqrt{n}}{n^2+1}$

   b. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{7\sqrt{n}}{n+1}$

   c. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{7n}{n^2+1}$

   d. all of the above

   e. none of the above

3. Find $D_x\left(x+\frac{3}{x}\right)^x$

a. $\left(\frac{x^2}{x^2+3}\right)\left(x+\frac{3}{x}\right)^x\left(1-\frac{3}{x^2}\right)$

b. $\left(\frac{-3}{x^2+3}\right)\left(x+\frac{3}{x}\right)$

c. $\left[\frac{x^2-3}{x^2+3}+\ln\left(x+\frac{3}{x}\right)\right]\left(x+\frac{3}{x}\right)^x$

d. $\left[\frac{-3}{x^2+3}+\ln\left(x+\frac{3}{x}\right)\right]\left(x+\frac{3}{x}\right)^x$

e. none of the above

4. Evaluate $\int_{-\pi/4}^{\pi}\sec^2 x dx$

a. $\sqrt{2}$

b. 1

c. –1

d. 2

e. none of the above

5. A parabolic satellite dish has a diameter of 5 feet and its depth at the center is 2 feet. Find the distance of its focus from the center.

   a. 18/25

   b. 10

   c. 4/5

   d. 25/32

   e. 5/4

6. Find $F'(x)$, if $F(x) = \int_{\sqrt{x}}^{x^2} \sqrt{t^2 - 1}\,dt$

   a. $2x\sqrt{x^2 - 1} - \dfrac{\sqrt{x^2 - 1}}{2\sqrt{x}}$

   b. $\sqrt{x^2 - 1} - \sqrt{x - 1}$

   c. $2x\sqrt{x^4 - 1} - \dfrac{\sqrt{x - 1}}{2\sqrt{x}}$

   d. 0

   e. none of the above

7. Let $y = \dfrac{-x^3 + 9x^2 - 17x - 3}{x - 3}$. What are the global extreme values of this function?

   a. no min; max is 10

   b. no max; min is 10

   c. no min; max is 3

   d. no max; min is 3

   e. there are no extreme values

8. Find $\frac{d^2y}{dx^2}$ if $y = e^{xy}$

a. y^2e^{xy}

b. $\frac{y^2}{1-xy}$

c. $(x+y)e^{xy}$

d. $\frac{3y^3-2xy^4}{(1-xy)^4}$

e. none of the above

9. Evaluate $D_x\left(\log_4(e^{\sec(8^x)})\right)$

a. $(8^x)\sec(8^x)\tan(8^x)$

b. $\frac{3}{2}(8^x)\sec(8^x)\tan(8^x)$

c. $2(8^x)\sec(8^x)\tan(8^x)$

d. $3(4^x)\sec(8^x)\tan(8^x)$

e. none of the above

10. You are standing at (1, 1, 3) on the surface $z = 3x^2 - 2xy + 2y^2$ If you move in the direction of greatest decrease in *z*, what will that rate of decrease be?

a. $\dfrac{-1}{2}$

b. $\dfrac{-\sqrt{5}}{2}$

c. $-2\sqrt{5}$

d. $-\sqrt{6}$

e. –2

11. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \dfrac{(2x-1)^n}{(-3)^n n}$

a. [-1, -2]

b. [-1, -2)

c. (-1, -2]

d. (-1, -2)

e. none of the above

12. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is $F = \dfrac{\mu W}{\mu \sin\theta + \cos\theta}$ where μ is a positive constant called the coefficient of friction and where $0 \le \theta \le \pi/2$. When is F minimized? Write your answer as a trigonometric function of θ.

    a. $\mu = \sin\theta$

    b. $\mu = \cos\theta$

    c. $\mu = \tan\theta$

    d. $\mu = \cot\theta$

    e. none of the above

13. Set up the definite integral (do not evaluate) used to find the first quadrant length of the graph given by the equation $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

    a. $\displaystyle\int_0^3 \frac{1}{3}\sqrt{\frac{81+7x^2}{9-x^2}}\,dx$

    b. $\displaystyle\int_0^3 \frac{1}{9}\sqrt{\frac{81+7x^2}{9-x^2}}\,dx$

    c. $\displaystyle\int_0^3 \frac{1}{3}\sqrt{\frac{81-5x^2}{9-x^2}}\,dx$

    d. $\displaystyle\int_0^2 \frac{1}{3}\sqrt{\frac{81-5x^2}{9-x^2}}\,dx$

    e. none of the above

14. Find the area of the triangle with vertices (3, 4, 1), (0, 2, 3), and (2, 2, -1).

    a. 100

    b. $78\sqrt{2}$

    c. 6

    d. $39\sqrt{2}$

    e. $\frac{3}{2}\sqrt{17}$

15. Find the lowest temperature on the plane $x + y + 2z = 1$ if temperature T is given by the formula $T = x^2 + y^2 + z^2$.

    a. 1/6

    b. 1/8

    c. 1

    d. ¼

    e. ½

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PART TWO: SHORT ANSWER QUESTIONS

16. Find the shortest distance between the point (2, 1, 3) and the surface $4X - 3Y + 1Z = 7$.

17. Compute the Riemann sum of f on the interval [0, 4], for $n = 4$, using the midpoint of each subinterval as the evaluation points, given $f'(x) = 2x$ and $f(0) = 1$.

18. What is the area bounded by the curves $f(x) = -(x-2)^2 + 4$ and $g(x) = x^2$?

19. What is the area of the region bounded by the polar equation $r = \csc\theta$ from π/4 to π/2?

20. Suppose that $f(x) = \begin{cases} \sqrt{4 - x^2} & 0 \le x \le 2 \\ -3\,|\,x - 3\,| + 3 & 2 \le x \le 5 \end{cases}$ and $g(x) = \int_0^x f(t)dt$. Find $f(3)$, $g(3)$, and $g'(3)$

21. Find the volume inside $X^2 + Y^2 + Z^2 = 4$ and also inside the cylinder $X^2 + Y^2 = 1$.

22. Find $\left(\dfrac{f \circ g}{2g}\right)'(3)$ given

x	***f(x)***	***g(x)***	***f′(x)***	***g′(x)***
3	-1	4	0	-1
4	1	3	2	7

23. A pilot is flying along a path whose coordinates are given by the following: $X = t^3 - 3$, $Y = t^2 + 1$, $Z = -t$, t = time. At t = 1, the pilot fires a missile straight ahead, along a tangent line toward a stationary object whose X coordinate is 7. Find the distance between the pilot and the object at the time the missile was fired.

24. Evaluate $\lim_{n\to\infty}\left(1 + \dfrac{3}{n}\right)^n$

25. Evaluate $\int_C 2xydx + 2x^2dy$ along curve C (oriented counterclockwise) where C is the boundary of the region bounded by $y = x^2$ and $y = x + 2$